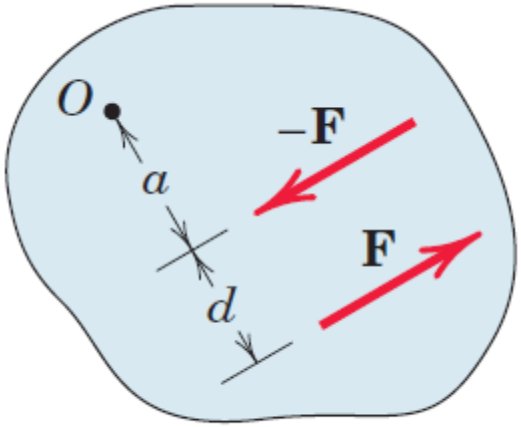
FORCE SYSTEMS

2/5 COUPLE

- The moment produced by two equal, opposite, and non-collinear forces is called a *couple*. Couples have certain unique properties and have important applications in mechanics.



(a)

$$M = F(a + d) - Fa$$

$$M = Fd$$

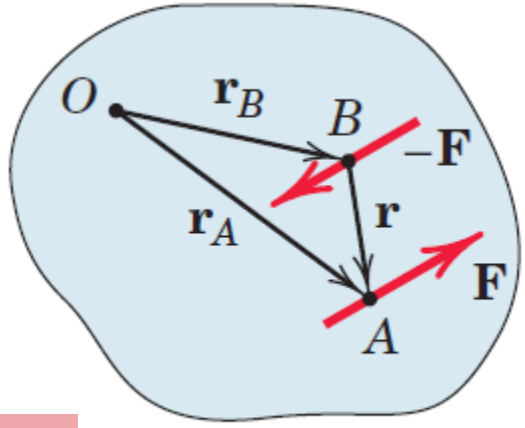
- **These two forces cannot be combined into a single force because their sum in every direction is zero.**
- **Their only effect is to produce a tendency of rotation.**

FORCE SYSTEMS

Vector Algebra Method

We may also express the moment of a couple by using vector algebra. With the cross-product notation of Eq. 2/6, the combined moment about point *O* of the forces forming the couple of Fig. 2/10*b* is

$$\mathbf{M} = \boldsymbol{r_A} \times \mathbf{F} + \boldsymbol{r_B} \times (-\mathbf{F}) = (\boldsymbol{r_A} - \boldsymbol{r_B}) \times \mathbf{F}$$



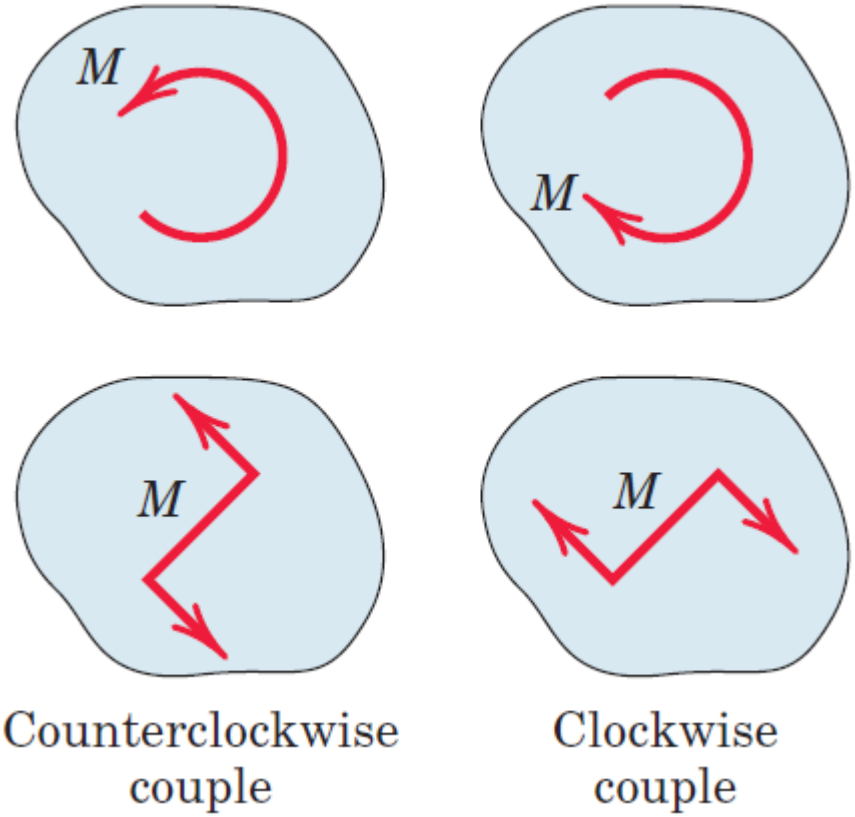
(*b*)

if the $\boldsymbol{r_A - r_B = r}$, we can express M as

$$\boldsymbol{M = r \times F}$$

FORCE SYSTEMS

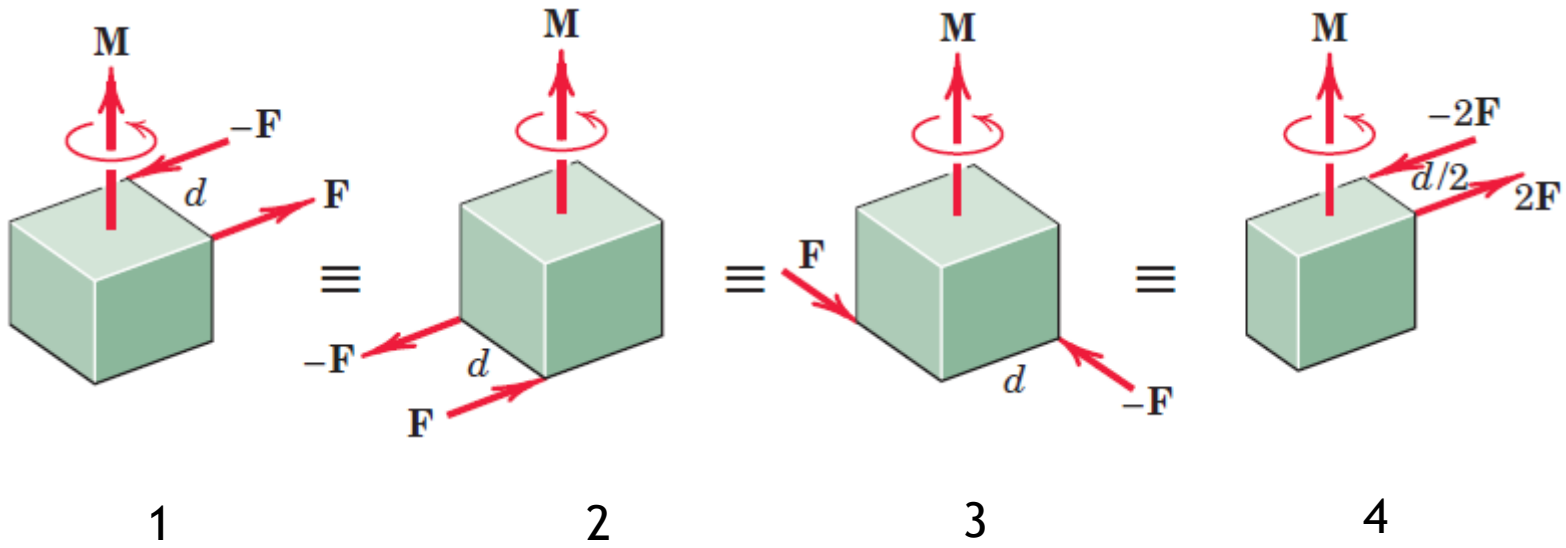
We can express a Couple as a figure below.



FORCE SYSTEMS

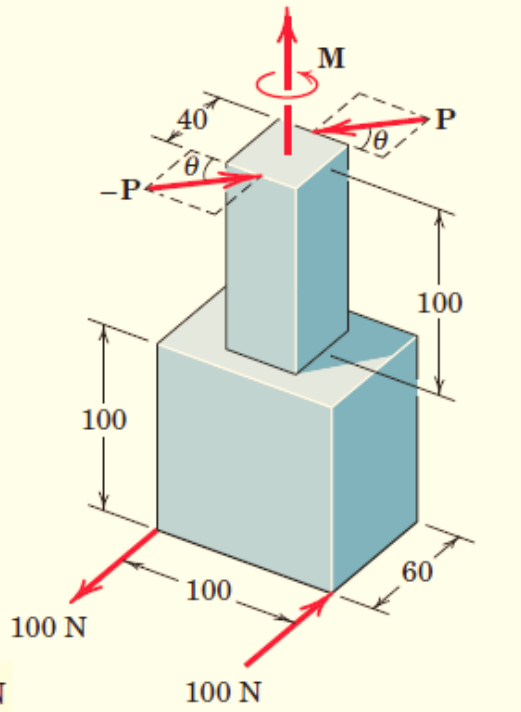
Equivalent Couples

Changing the values of ***F*** and ***d*** does not change a given couple as long as the product ***F.d*** remains the same. Likewise, a couple is not affected if the forces act in a different but parallel plane.



FORCE SYSTEMS

Ex:-The rigid structural member is subjected to a couple consisting of the two **100-N** forces. Replace this couple by an equivalent couple consisting of the two forces **P and -P**, each of which has a magnitude of **400 N**. Determine the proper angle θ .



Solution

The original couple is counterclockwise when the plane of the forces is viewed from above, and its magnitude is

$$M = F \times d = 100 \times (0.1) = 10 \text{ N.m}$$

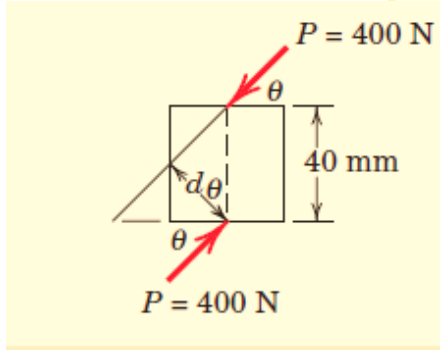
The forces P and –P produce a counterclockwise couple

$$M = 400\,(0.040)\cos\theta$$

Equating the two expressions gives

$$10 = 400\,(0.040)\cos\theta$$

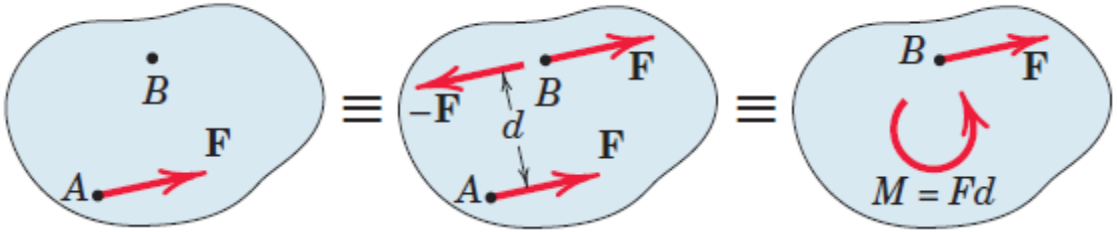
$$\theta = \cos^{-1}\frac{10}{16} = 51.3$$



FORCE SYSTEMS

Resolution of a force into a force and a couple

In sometime becomes necessary to replace a force action at a given point by equal force acting through some other point. This introduces a couple



FORCE SYSTEMS

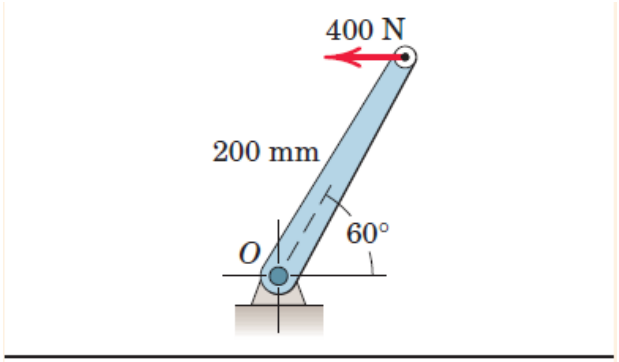
Ex:- Replace the horizontal **400-N** force acting on the lever by an equivalent system consisting of a force at **O** and a couple.

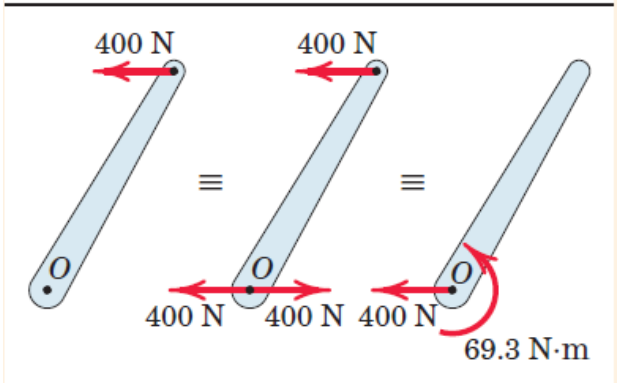
Solution.

We apply two equal and opposite **400-N** forces at ***O*** and identify the counterclockwise couple

$$M = F \times d$$

$$M = 400\ (0.200 \sin 60) = 69.3\ \text{N.m}$$



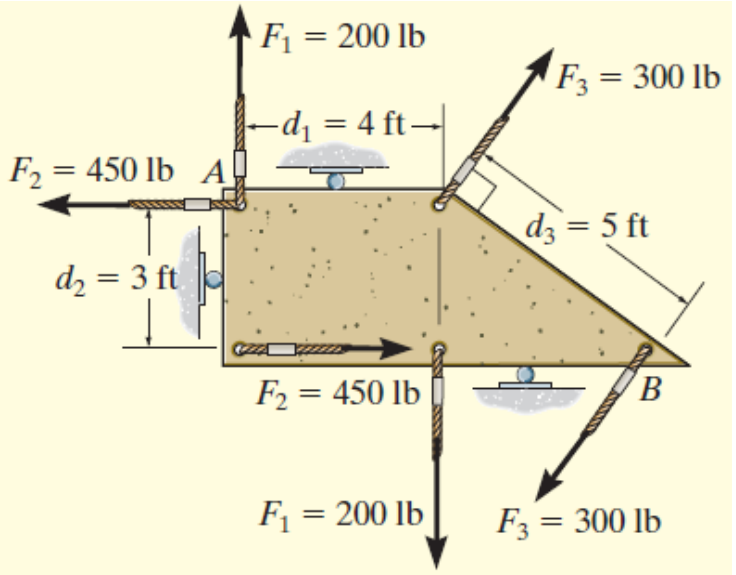


FORCE SYSTEMS

Ex:- Determine the resultant couple moment of the three couples acting on the plate in figure below.

As shown the perpendicular distances between each pair of couple forces are

d1 = 4 ft., d2 = 3 ft., d3 = 5 ft.



$$\circlearrowleft + M_R = \Sigma M;\ M_R = -F_1 d_1 + F_2 d_2 - F_3 d_3$$

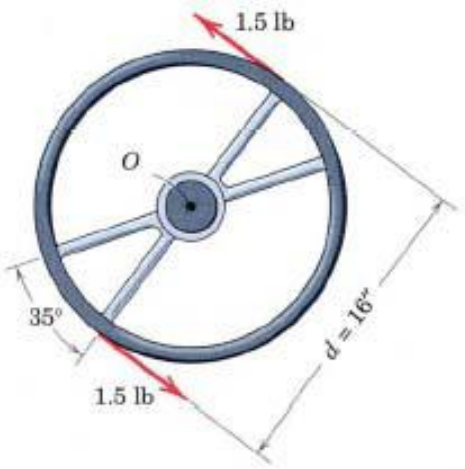
$$= (-200\ \text{lb})(4\ \text{ft}) + (450\ \text{lb})(3\ \text{ft}) - (300\ \text{lb})(5\ \text{ft})$$

$$= -950\ \text{lb}\cdot\text{ft} = 950\ \text{lb}\cdot\text{ft}\ \circlearrowright \qquad \textit{Ans.}$$

The negative sign indicates that MR has a clockwise rotational sense.

FORCE SYSTEMS

EX:- Determine the moment associated with the forces shown in fig.

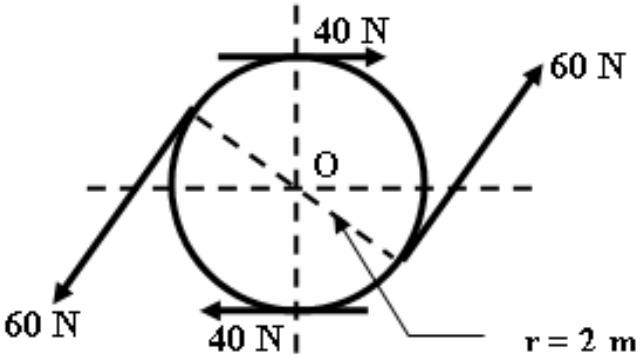


Solution

Mo = F * d

= 1.5 * 16 = 24 lb .in

Ex:- Compute the magnitude and direction of the resultant couples action on the body shown



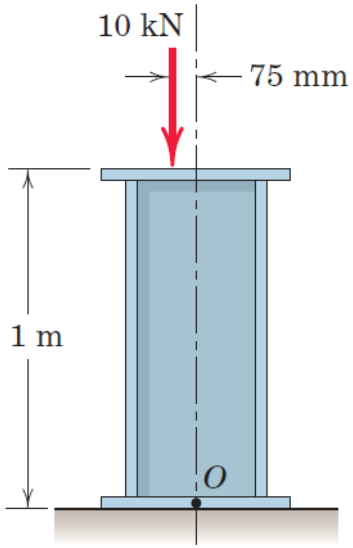
+ Mo = 60 * 4 – 40 * 4

= 240 – 160 = 80 N .m

FORCE SYSTEMS

H.W

Q1:- Replace the **10-kN** force acting on the steel column by an equivalent force-couple system at point ***O***. This replacement is frequently done in the design of structures.



Q2:- The 180-N force is applied to the end of body *OAB*. If $\theta = 50$, determine the equivalent force–couple system at the shaft axis ***O***.